BOARD QUESTION PAPER : MARCH 2015
MATHEMATICS AND STATISTICS

Time: 3 Hours **Total Marks: 80**

Note:

i. All questions are compulsory.
ii. Figures to the right indicate full marks.
iii. Graph of L.P.P. should be drawn on graph paper only.
iv. Answer to every new question must be written on a new page.
v. Answers to both sections should be written in the same answer book.
vi. Use of logarithmic table is allowed.

SECTION – I

Q.1. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions: **(6)[12]**

i. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $A^6 =$ _______.

(A) 6A (B) 12A (C) 16A (D) 32A

ii. The principal solution of $\cos^{-1}\left(-\frac{1}{2}\right)$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{2}$

iii. If an equation $hxy + gx + fy + c = 0$ represents a pair of lines, then
(A) fg = ch (B) gh = cf (C) fh = cg (D) hf = – cg

(B) Attempt any THREE of the following: **(6)**

i. Write the converse and contrapositive of the statement-
"If two triangles are congruent then their areas are equal."

ii. Find 'k', if the sum of slopes of lines represented by equation $x^2 + kxy - 3y^2 = 0$ is twice their product.

iii. Find the angle between the planes $\bar{r}\cdot\left(2\hat{i}+\hat{j}-\hat{k}\right) = 3$ and $\bar{r}\cdot\left(\hat{i}+2\hat{j}+\hat{k}\right) = 1$.

iv. The cartesian equations of line are $3x - 1 = 6y + 2 = 1 - z$. Find the vector equation of line.

v. If $\bar{a} = \hat{i}+2\hat{j}$, $\bar{b} = -2\hat{i}+\hat{j}$, $\bar{c} = 4\hat{i}+3\hat{j}$, find x and y such that $\bar{c} = x\bar{a} + y\bar{b}$.

Q.2. (A) Attempt any TWO of the following: **(6)[14]**

i. If A, B, C, D are (1, 1, 1), (2, 1, 3), (3, 2, 2), (3, 3, 4) respectively, then find the volume of the parallelopiped with AB, AC and AD as the concurrent edges.

ii. Discuss the statement pattern, using truth table: $\sim(\sim p \wedge \sim q) \vee q$

iii. If point $C(\bar{c})$ divides the segment joining the points $A(\bar{a})$ and $B(\bar{b})$ internally in the ratio m : n, then prove that $\bar{c} = \frac{m\bar{b}+n\bar{a}}{m+n}$.

(B) Attempt any TWO of the following: (8)

i. Find the direction cosines of the line perpendicular to the lines whose direction ratios are $-2, 1, -1$ and $-3, -4, 1$.

ii. In any ΔABC, if a^2, b^2, c^2 are in arithmetic progression, then prove that cot A, cot B, cot C are in arithmetic progression.

iii. The sum of three numbers is 6. When second number is subtracted from thrice the sum of first and third number, we get number 10. Four times the third number is subtracted from five times the sum of first and second number, the result is 3. Using above information, find these three numbers by matrix method.

Q.3. (A) Attempt any TWO of the following: (6)[14]

i. If θ is the acute angle between the lines represented by equation $ax^2 + 2hxy + by^2 = 0$, then prove that $\tan\theta = \left|\dfrac{2\sqrt{h^2 - ab}}{a + b}\right|$, $a + b \neq 0$.

ii. If the lines $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-1}{4}$ and $\dfrac{x-3}{1} = \dfrac{y-k}{2} = \dfrac{z}{1}$ intersect each other, then find the value of 'k'.

iii. Construct the switching circuit for the following statement:
$[p \vee (\sim p \wedge q)] \vee [(\sim q \wedge r) \vee \sim p]$

(B) Attempt any TWO of the following: (8)

i. Find the general solution of $\cos x - \sin x = 1$.

ii. Find the equations of the planes parallel to the plane $x - 2y + 2z - 4 = 0$, which are at a unit distance from the point (1, 2, 3).

iii. A diet of a sick person must contain at least 48 units of vitamin A and 64 units of vitamin B. Two foods F_1 and F_2 are available. Food F_1 costs ` 6 per unit and food $F_2$ costs ` 10 per unit. One unit of food F_1 contains 6 units of vitamin A and 7 units of vitamin B. One unit of food F_2 contains 8 units of vitamin A and 12 units of vitamin B. Find the minimum cost for the diet that consists of mixture of these two foods and also meeting the minimal nutritional requirements.

SECTION – II

Q.4. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions: (6)[12]

i. A random variable X has the following probability distribution:

$X = x$	-2	-1	0	1	2	3
$P(x)$	0.1	0.1	0.2	0.2	0.3	0.1

Then $E(x) =$

(A) 0.8 (B) 0.9
(C) 0.7 (D) 1.1

ii. If $\int_0^{\alpha} 3x^2\, dx = 8$, then the value of α is

(A) 0 (B) -2
(C) 2 (D) ± 2

iii. The differential equation of $y = \frac{c}{x} + c^2$ is

(A) $x^4\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = y$ (B) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(C) $x^3\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} = y$ (D) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$

(B) Attempt any THREE of the following: **(6)**

i. Evaluate: $\int e^x \left[\frac{\sqrt{1-x^2}\cdot\sin^{-1}x + 1}{\sqrt{1-x^2}}\right] dx$

ii. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots\infty}}}$, then show that $\frac{dy}{dx} = \frac{\cos x}{2y-1}$

iii. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos x}\, dx$

iv. If $y = e^{ax}$, show that $x\frac{dy}{dx} = y \log y$.

v. A fair coin is tossed five times. Find the probability that it shows exactly three times head.

Q.5. (A) Attempt any TWO of the following: **(6)[14]**

i. Integrate: $\sec^3 x$ w.r.t. x.

ii. If $y = (\tan^{-1}x)^2$, show that

$(1 + x^2)^2 \frac{d^2y}{dx^2} + 2x(1 + x^2)\frac{dy}{dx} - 2 = 0$

iii. If $f(x) = \left[\tan\left(\frac{\pi}{4} + x\right)\right]^{\frac{1}{x}}$, for $x \neq 0$

$= k,$ for $x = 0$

is continuous at $x = 0$, find k.

(B) Attempt any TWO of the following: **(8)**

i. Find the co-ordinates of the points on the curve $y = x - \frac{4}{x}$, where the tangents are parallel to the line $y = 2x$.

ii. Prove that: $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

iii. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx$

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. Find a and b, so that the function $f(x)$ defined by

$f(x) = -2\sin x,$ for $-\pi \le x \le -\frac{\pi}{2}$

$= a\sin x + b,$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$= \cos x,$ for $\frac{\pi}{2} \le x \le \pi$ is continuous on $[-\pi, \pi]$.

ii. If $\log_{10}\left(\dfrac{x^3 - y^3}{x^3 + y^3}\right) = 2$, then show that $\dfrac{dy}{dx} = -\dfrac{99x^2}{101y^2}$.

iii. Let the p.m.f. (probability mass function) of random variable x be

$P(x) = \binom{4}{x}\left(\dfrac{5}{9}\right)^x\left(\dfrac{4}{9}\right)^{4-x}$, $x = 0, 1, 2, 3, 4$.

$= 0$, otherwise

Find $E(x)$ and $Var(x)$.

(B) Attempt any TWO of the following: (8)

i. Examine the maxima and minima of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$. Also, find the maximum and minimum values of $f(x)$.

ii. Solve the differential equation $(x^2 + y^2)dx - 2xydy = 0$.

iii. Given the p.d.f. (probability density function) of a continuous random variable x as:

$f(x) = \dfrac{x^2}{3}$, $-1 < x < 2$

$= 0$, otherwise

Determine the c.d.f. (cumulative distribution function) of x and hence find $P(x < 1)$, $P(x \le -2)$, $P(x > 0)$, $P(1 < x < 2)$.