Digital Communication Unit-III
Stochastic Process

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- Unwanted signals: Noise

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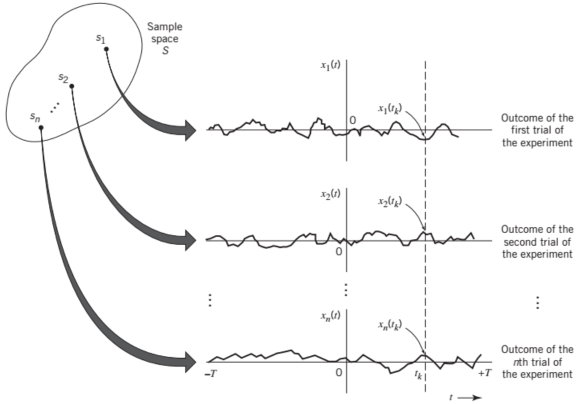

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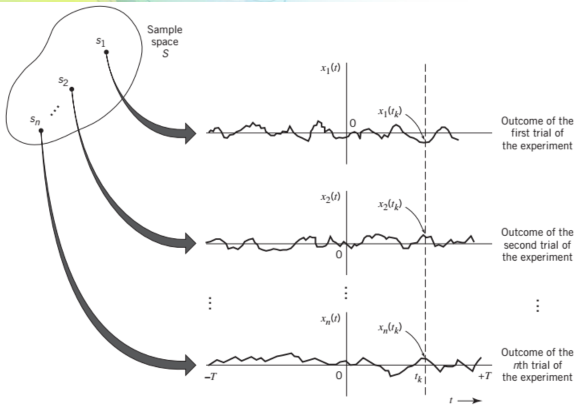

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    - $|x_j(t)|, j = 1, 2, \ldots, n$

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  - "A stochastic process X(t) is an ensemble of time functions, which, together with a probability rule, assigns a probability to any meaningful event associated with an observation of one of the sample functions of the stochastic process".

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- Summary of Random Processes
  - Wide-Sense Stationary processes
    - Strictly Stationary Processes
    - Ergodic Processes

- Properties of Strictly Stationary Process:
  - For k = 1, we have $F_{X(t)}(x) = F_{X(t+\tau)}(x) = F_X(x)$ for all t and τ. *First-order distribution function of a strictly stationary stochastic process is independent of time t.*
  - For k = 2, we have $F_{X(t_1),X(t_2)}(x_1, x_2) = F_{X(0),X(t_1-t_2)}(x_1, x_2)$ for all t_1 and t_2. *Second-order distribution function of a strictly stationary stochastic process depends only on the time difference between the sampling instants and not on time sampled.*
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- Summary of Random Processes
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Mean, Correlation, and Covariance Functions of WSP

- Mean:

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    - The autocovariance function of a weakly stationary process $X(t)$ depends only on the time difference $(t_2 - t_1)$
    - The mean and autocorrelation function only provide a weak description of the distribution of the stochastic process $X(t)$.

Examples

Show that the Random Process $X(t) = Acos(\omega_c t + \Theta)$ is wide sense stationary process, where Θ is RV uniformly distributed in range $(0, 2\pi)$

Answer:

- The ensemble consist of sinusoids of constant amplitude A and constant frequency ω_c, but phase Θ is random.

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$$= 0, elsewhere$$

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- Thus the ensemble mean of sample function amplitude at any time instant t is zero.
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Answer:

- Continue. . .

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- From $\overline{X(t)} = 0$ and $R_{XX}(\tau) = \frac{A^2}{2} cos(\omega_c(\tau))$ it is clear that $X(t)$ is Wide Sense Stationary Process

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    - where μ_X and $R_{XX}(\tau)$ are the ensemble averages of the same random process.

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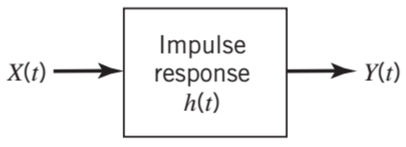

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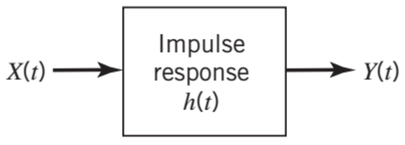

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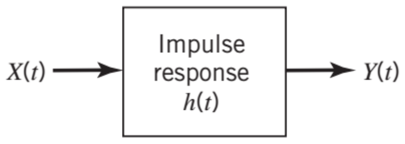

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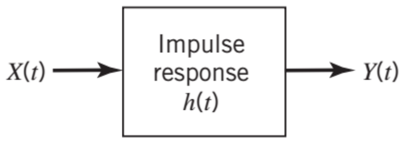

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Transmission of a Weakly Stationary Process through a LTI Filter

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    - $$M_{YY}(t,u) = E\left[Y(t)Y(u)\right]$$

      where t and u denote two values of the time at which the output process $Y(t)$ is sampled

      $$. = E\left[\int_{-\infty}^{+\infty} h(\tau_1)X(t-\tau_1)d\tau_1 \int_{-\infty}^{+\infty} h(\tau_2)X(u-\tau_2)d\tau_2\right]$$

      Here again, provided that the mean-square value $E\left[X^2(t)\right]$ is finite for all t and the filter is stable

      $$= \int_{-\infty}^{+\infty}\left\{h(\tau_1)\int_{-\infty}^{+\infty} d\tau_2 h(\tau_2)E\left[X(t-\tau_1)X(u-\tau_2)\right]\right\}d\tau_1$$

      $$= \int_{-\infty}^{+\infty}\left[h(\tau_1)\int_{-\infty}^{+\infty} d\tau_2 h(\tau_2)M_{XX}(t-\tau_1, u-\tau_2)\right]d\tau_1$$

      When the input $X(t)$ is a weakly stationary process, the autocorrelation function of $X(t)$ is only a function of the difference between the sampling times $t\tau_1$ and $u\tau_2$. Thus, putting $\tau = ut$

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which, of course, is a constant.

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$$h(\tau_1) = \int_{-\infty}^{+\infty} H(f)\exp(j2\pi f\tau_1)df$$

$$E\left[Y^2(t)\right] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(\tau_1)h(\tau_2)R_{XX}(\tau_1-\tau_2)d\tau_1 d\tau_2$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} H(f)\exp(j2\pi f\tau_1)df\right] h(\tau_2)R_{XX}(\tau_1-\tau_2)d\tau_1 d\tau_2$$

$$= \int_{-\infty}^{+\infty}\left[H(f)\int_{-\infty}^{+\infty} h(\tau_2)d\tau_2\int_{-\infty}^{+\infty} R_{XX}(\tau_1-\tau_2)\exp(j2\pi f\tau_1)d\tau_1\right] df$$

by $\tau = \tau_1 - \tau_2$

$$= \int_{-\infty}^{+\infty} H(f)\left[\int_{-\infty}^{+\infty} h(\tau_2)\exp(j2\pi f\tau_2)d\tau_2\int_{-\infty}^{+\infty} R_{XX}(\tau)\exp(-j2\pi f\tau)d\tau$$

$$= \int_{-\infty}^{+\infty}|H(f)|^2\left[\int_{-\infty}^{+\infty} R_{XX}(\tau)\exp(-j2\pi f\tau)d\tau\right] df$$

as $|H(f)|^2 = H(f)H^*(f)$ and square brackets represents fourier transform of the autocorrelation function R_{XX} of the input process

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- The mean-square value of the output of a stable linear time-invariant filter in response to a weakly stationary process is equal to the integral over all frequencies of the power spectral density of the input process multiplied by the squared magnitude response of the filter.

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Properties of the Power Spectral Density(PSD)

- Properties of PSD

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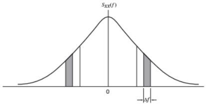

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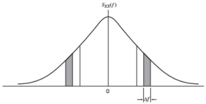

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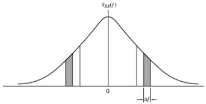

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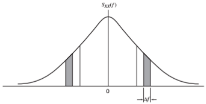

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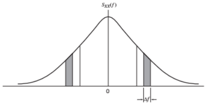

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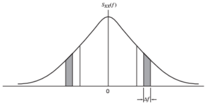

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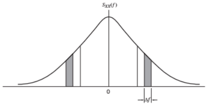

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$$P_{XX}(f) = \frac{S_{XX}(f)}{\int_{-\infty}^{\infty} S_{XX}(f)df}$$

The Gaussian Process

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     - If the random variables $X(t_1), X(t_2), \ldots, X(t_n)$, obtained by respectively sampling a Gaussian process $X(t)$ at times $t_1, t_2, \ldots, t_n$, are uncorrelated, that is

$$E\left[(X(t_k) - \mu_{X(t_k)})(X(t_i) - \mu_{X(t_i)})\right] = 0 \qquad i \neq k$$

then these random variables are statistically independent.

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  - The noise analysis of communication systems is based on a source of noise called white-noise, which is to be discussed next.

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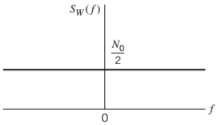

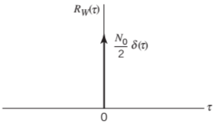

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